GROWTH OPPORTUNITIES AND ELEMENTARY REAL OPTIONS

Andrea Mantovi

Dipartimento di Economia

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Growth opportunities: assets *not* in place

Profitability of the investment opportunity:
net present value of the asset once *in* place \(\rightarrow\) PVGO

going concern

There may exist **flexibility** in the investment opportunity

The **value** of flexibility:

“DCF analysis does not reflect the value of *management*”
(Brealey and Myers, 2003, p. 617)

value of flexibility \(\leftrightarrow\) **optimality** of the investment exercise
A global industrial company can better capture opportunities

Source: Global Insight, Monthly Forecast Update, Jan. ’09
“ciò che è rigido è destinato a soccombere
ciò che è flessibile è destinato a crescere”

Lao Tsu, Tao Te Ching
The **ownership** of an opportunity to invest can be given a **value** which “depends on the **rule** for deciding whether the options are to be exercised” (Myers, 1977, p. 149)

**Real Options (RO)**

In the last two decades the development of a coherent approach to the PVGO and the value of flexibility with respect to the optimality of exercising the investment opportunity

**RO models are meant to yield basic **insights** in a consistent (possibly comprehensive) framework**
**RO: a method**

- risk neutral valuation
- stochastic underlying
- optimization perspective of **dynamic programming**
- parallel to **financial call options**
- a **bridge** between investment science and optimal control

**Applications:**

- general competitive **equilibrium** [Dixit and Pindyck, 1994, ch.8]
- regulation of imperfectly competitive markets [Roques and Savva, 2009]
RO are meant to define the value of flexibility under uncertainty continuous time: brownian motions

The elementary RO in continuous time:
the value of waiting to invest (McDonald and Siegel, 1986)
asset: GBM with positive drift, liability: constant lump sum expenditure
call it RO₁

Standard expositions (Dixit and Pindyck, 1994)
are mainly issue driven: the problems under inquiry tailor the approach (entry and exit decision, sequential investment, etc.)
built upon RO₁

The shaping of a theory is structure driven:
a RO theory should encompass the complete realm of flexibility under uncertainty
Standard approaches assess the value of flexibility on the asset side of the investment opportunity.

The underlying upon which the RO is defined is the stochastic value of the asset, typically represented by a geometric brownian motion with positive drift.

In fact, flexibility may stem from a decreasing investment expenditure.

Can we build an orthodox RO model upon such an intuitive insight?
Flexibility and value in a deterministic setting
the value of waiting to invest

Dixit and Pindyck (1994, ch. 5) call it DO₁

 Asset \[ S(t) = V e^{\alpha t} \]
   value driver on the asset side

 Liability \[ I \]

 Profitability measures
\[
P(t) = (V e^{\alpha t} - I) e^{-rt} \\
P(S) = S - I
\]

\[ 0 < \alpha < r \]

 the optimization problem yields a unique solution
Flexibility and value in a deterministic setting
a dual model of waiting to invest
call it DO$_3$

Asset  \( S = \Xi \) constant value

Liability  \( I e^{-\alpha t} \)
value driver on the liability side

Profitability measures
\[
P(t) = (\Xi - I e^{-\alpha t}) e^{-r t}
\]
\[
P(S) = \xi S - I \quad \xi > 1
\]

\( \alpha, r > 0 \)
unique solution
Dual problems

Asymptotic behaviour: independence

Closed form solution: adapting basic models to a ‘covering’ of the space of opportunities

**Standard** RO investment trigger:
the **multiplier** for the **break even** trigger

\[ y^* = \frac{b}{b+1} \frac{1}{\xi} = \frac{b}{b+1} y_{\text{break even}} \]

\( b \) : ratio between risk free discount rate and ‘growth’ rate

the picture remains in the stochastic setting:
well defined deterministic limit: a key issue on the RO approach
Ito’s lemma generalizes the chain rule of calculus to the stochastic setting.

Elementary RO models are represented by smooth differential problems. On the other hand, sample paths of Brownian motions are not differentiable.

The structure of elementary RO models:
- second order (ordinary or partial) differential equation
- free boundary problem
Contingent claims valuation

Law of one price $\leftrightarrow$ No arbitrage $\rightarrow$ Hedging

A call option $V$ as a contingent claim on a stochastic underlying asset $S$
so as to hedge a portfolio consisting of $S$ and $V$

Financial options: Black and Scholes (1973), Merton (1973)

The stochastic underlying has been usually taken to be
the value of the asset

In the model $RO_3$ the stochastic underlying is the expenditure
new perspective on contingent claims?
differential equation

\[ \frac{1}{2} \sigma^2 y^2 \frac{d^2 W}{dy^2} - \alpha y \frac{dW}{dy} - rW = 0 \]

2d linear space of solutions

power laws determined by the fundamental quadratic

\[ W(y) = A_1 y^{\beta_1} + A_2 y^{\beta_2} \]

boundary/initial condition

\[ W(y) \rightarrow 0 \quad y \rightarrow +\infty \]

value matching

\[ W(y^*) = A_1 y^{*\beta_1} = \Xi(1 - \xi y^*) \]

smooth pasting

\[ \frac{dW}{dy}(y^*) = A_1 \beta_1 y^{*\beta_1 - 1} = -\Xi \xi \]
In between investment and disinvestment

RO have been employed to account for optimal disinvestment

Underlying: GBM with negative drift $-\alpha$

closed form solution

$$S(t) = \exp \left( -\alpha t - \frac{\sigma^2 t}{2} \right) \exp(\sigma z(t))$$

mean $E(S(t)) = \exp(-\alpha t)$ (green line)
In between investment and disinvestment

Positive power law: standard investment problems

insight: the value of the RO vanishes at $S = 0$

underlying: GBM with positive drift (value driver on the asset side)

\[ W(y) = A_1 y^{\beta_1} \]

Negative power law: disinvestment

(Lambrecht and Myers, 2007)

insight: the value of the RO vanishes at $y = \infty$

underlying: GBM with negative drift (value driver on the liability side)

$RO_3$

investment: increasing profitability $\leftarrow$ diminishing expenditure

finite asymptotic limit $\rightarrow$ negative power law

analogy with disinvestment
**RO$_3$: comparative statics for the relevant value drivers**

The larger the ‘growth’ rate $\alpha$
the closest the trigger and the higher the value of the RO.

$$\frac{\partial W}{\partial \alpha} = \frac{\partial W}{\partial y^*} \frac{\partial y^*}{\partial \alpha} > 0$$

$$\frac{\partial W}{\partial \sigma} = \frac{\partial W}{\partial y^*} \frac{\partial y^*}{\partial \sigma} < 0$$

The larger the volatility the farther the trigger
and the lower the value of the option

**RO$_3$: the size of shocks diminishes**
as probability approaches its asymptotic limit

“the firm requires a higher return to invest when volatility is higher, but it does so exactly because it is more likely to encounter periods of very low returns”

(Dixit and Pindyck, 1994, p. 422)

consistency with standard RO models
Elementary dual real options

$RO_1$: the value of the asset not in place is a GBM with positive drift
investment expenditure: a constant lump sum

$RO_2$: the value of the asset not in place is a constant
investment expenditure: stochastic lump sum, GBM with negative drift

**Elementary** stylized fact: minimal investment problem

**Duality**: value drivers solely on the asset ($RO_1$) or liability ($RO_2$) side

A way towards a theory?

Should the model $RO_2$ turn out to be **wrong** in some sense
a step forward would anyway come about
assessing definite **boundaries** for the RO approach
Capacity choice as incremental investment
(Dixit and Pindyck, 1994, ch. 11)

A paradigmatic application of the RO structure
capacity increase as an iterated investment problem:
at every instant the firm invests a lump sum to increase the stock of capital
thereby acquiring a ‘marginal project’

Bellman equation: profit maximization under stochastic demand
underlying on the asset side

Investment expenditure and or cost measures as underlying and value drivers?

Duality $\text{RO}_1 - \text{RO}_3$ : organizing insights
Applications

Inside the firm: **optimal contracts**
Grenadier and Wang (2005): RO₁ under adverse selection and moral hazard
Schianchi and Mantovi (2007): corporate governance,
going concern and incentives

Outside the firm: **imperfect competition**
Smit and Trigeorgis (2004) tailor natural lines
along which RO can be made to fit Industrial Economics

games and options do fit naturally inasmuch RO define payoffs

issue: solution concepts and numerical values of payoffs
a somewhat vague principle

“economic problems are also economic opportunities”
(Dixit, 2004, Lawlessness and Economics, Princeton)

and a somewhat vague conjecture

once economic opportunities encompass flexibility and stochasticity
a real option can be employed
to define the value of the opportunity
in connection with optimal exercise